

## Shadaj and Taar Shadaj: Similar yet Unique

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### Abstract

*Shadaj is considered as the fundamental pillar of Raag-Sangeet. It is the reference note that gives birth to various other musical notes. Among these musical notes, a very special note that is double the frequency of Shadaj is also called Shadaj (Tar). The reason for this nomenclature is the audible similarity of these two notes. Despite this similarity, they hold subjective musical relationships with any other note. The present study is an analytical approach to highlight the similarity and differences in the musical nature of Shadaj and Tar Shadaj. The analysis concludes that the two Shadajs appear analogous to identical twins that are genetically the same, but subjectively responsive to the same environment.*

**Keywords:** Shadaj, Octave, Musical space, Frequency

### 1. Introduction

The human auditory field corresponds to a specific range of frequencies, i.e., 20- 20,000 Hz. Within this range (bound by vocal/instrumental constraints), a musician holds immense freedom to combine various frequency levels (aesthetically) and weave a musical ambiance. Before beginning this musical weave, the selection of a reference point is the primary requirement. But why? Just like one needs a reference point in space to comprehend the amount of distance progressed, a reference point in time to gauge the amount of time elapsed, one needs a reference in music to establish and comprehend the musical space. This reference is nomenclated as “Shadaj” (or Sa) in Indian music.

An interesting point to ponder here is whether all the frequency levels (forming a continuous spectrum) that succeed or precede the Shadaj are musically relevant or only some discrete levels? The idea is analogous to the colours in a rainbow, which are distinctly classified as seven in number but are present in various shades—one merging into another. In the present study, we refrain from venturing into the domain of the “shades of musical notes” or the number of possible musically relevant frequency levels because the point that we wish to highlight holds true irrespective of the number of possible musical frequencies (or Shrutis) in an octave. The frequency step that is twice the Shadaj (or to put it simply, the 8<sup>th</sup> note that succeeds Sa, Re, Ga, Ma, Pa, Dha, Ni) is a very special frequency level and is also termed as Shadaj (known as Taar Shaaj or Sa). The article is an effort to highlight the reason for “the similarity in the nomenclature” and yet “the differences in the characters” of Shadaj and Taar Shadaj, analytically. Various fundamental concepts, namely, octave formation, the difference between

musical space and numerical space, etc. that serve as background for the study, are also presented.

## 2. Shadaj and Tar Shadaj: Similar

It is important to question here, why the reference note is termed as Shadaj and not something else? The answer to this question is explored in section 4.1. For now, we focus our attention on why the 8<sup>th</sup> note after Sa, Re, Ga, Ma, Pa, Dha, and Ni is also named as same as the reference note.

Having fixed a reference note/Shadaj, as one starts the musical ascent (Shadaj being continuously played in the backdrop), experiences different musical ambiances at different frequency levels, as an effect of “conversation” with Shadaj. The experience keeps altering with the rise from one frequency level to another and highlights certain “conversations” as musically pleasant over the others. On continuing the ascent, one lands at a very special point that introduces a unique “conversational experience” with respect to the reference point. This point is unique, for it does not register an audible impression of landing at a different frequency level but the Shadaj itself. The sound appears accurately blended with the Shadaj, and the ear perceives a Shadaj at this frequency.

Even before the quantification of this frequency level became possible, our musical forefathers knew that it was a different frequency level, higher than the reference note. However, despite knowing that this was a different frequency level, their auditory judgement convinced them to nomenclate it the same as the reference note, and hence the 8<sup>th</sup> note after Sa, Re, Ga, Ma, Pa, Dha, Ni was termed as Sa.

This decision appears quite justifiable because unless the conversation of a note and the reference note has something new to highlight, it is not worth designating an individuality to that frequency level. Quantitatively, this Shadaj (Taar) is double the frequency of the reference chosen (Shadaj).

This brings forth a very important concept of musical periodicity between the musical notes, briefly explained in the following section.

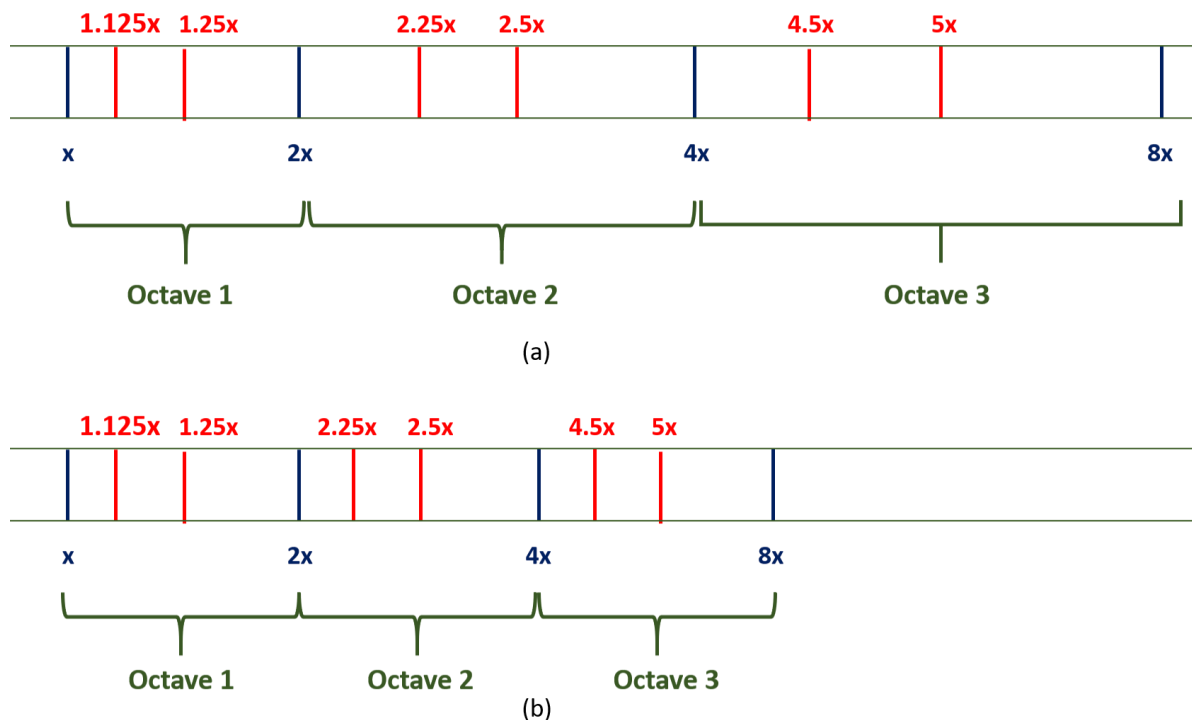
## 3. Musical space and Numerical space

The periodicity in the musical nature of frequency levels is a generalized phenomenon, observable for all the frequency levels that are  $2^n$  levels apart (where  $n$  is an integer), e.g., the frequency level  $x$  is audibly similar to the frequency levels at  $2x$ ,  $4x$ ,  $8x$ ,  $16x$  and so on.

The interval between two musically periodic frequencies (let say Shadaj and Tar Shadaj) is termed as an “Octave.” If  $x$  is the reference/Shadaj, the endpoints of successive octaves occur at  $2x$ ,  $4x$ ,  $8x$ ,  $16x$ , and so on. An interesting feature of the octaves is that the number of musical frequencies in all the octaves is equal, irrespective of the size of the octave. Intuitionally, the number of musical frequencies should increase as the span of an octave increase. However, it is essential to note that although the “numerical space” of various Octaves may be different, their “musical space” is the same, as shown in Fig. 1. This is because the musically relevant frequencies of an octave are related by ratios and not by the numerical distances between them. For example, if we consider the first two musical frequencies in an octave occur at 1.125 and 1.25 times of the reference note  $x$ , the musically relevant frequencies in octave 2 will be found

at the same ratios w.r.t starting point ( $2x$ ) of the octave 2, i.e., at  $2.25x$  and  $2.5x$ . Similarly, for the other octaves.

Thus, the musical ratios are retained in all the octaves and not the numerical spaces between them. This forms an important basis for further discussion on the distinct nature of Shadaj and Tar Shadaj.



**Fig. 1.** (a) Numerical space, and (b) Musical space

#### 4. Shadaj and Tar Shadaj: Distinct

This section highlights the individualities of these two notes. They are different with respect to their “musical conversations” with other musical notes (namely Re, Ga, Ma, Pa, Dha, Ni). But, let us first analytically explore the birth of these six “other musical notes.”

##### 4.1. Shat-janm: Birth of six musical frequencies

Imagine a string tensed at both its ends to produce a certain reference frequency (say 240 Hz). When plucked, one gets to hear several other frequencies simultaneously, along with the reference frequency (240 Hz). These frequencies are colloquially termed as overtones or harmonics.

When the string vibrates in its full length, the frequency produced is termed as the fundamental frequency (240 Hz). When it vibrates in two halves, the frequency produced is double the fundamental tone (480 Hz) and called the first overtone. Similarly, when the string vibrates in three equal parts, the frequency produced is thrice the fundamental (720 Hz) and is called the second overtone. This continues with the production of third (960 Hz), fourth (1200 Hz), fifth (1440 Hz) overtones, and so on.

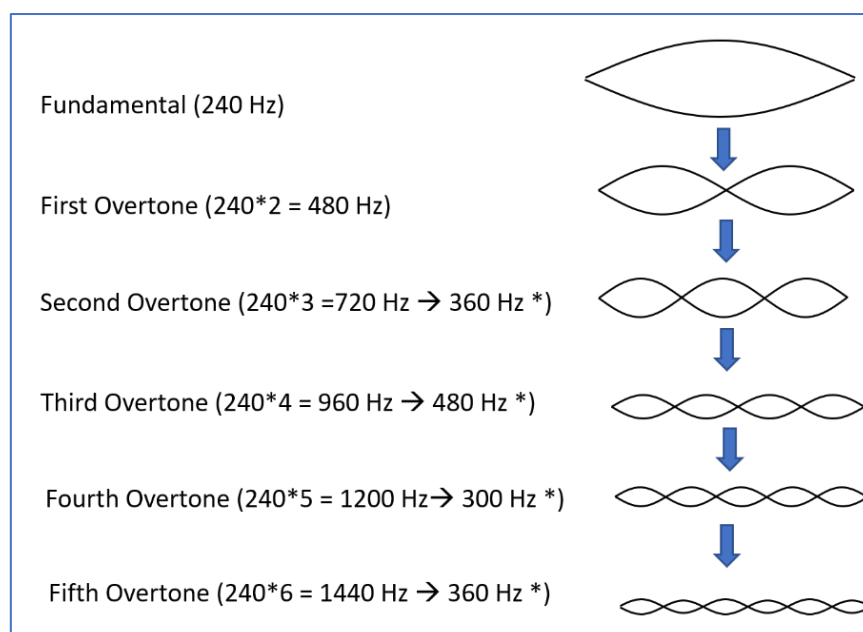
Are these overtones equally loud and discernible? These overtones do not have equal amplitudes. Fundamental overtone is the loudest, followed by first, second, third, fourth, fifth, and sixth harmonics. The sound becomes practically inaudible beyond 5-6 overtones.

An interesting phenomenon here is the interpretation of these overtones by the human brain. Since the reference chosen is 240 Hz, the upper limit of the octave gets fixed by 480 Hz. This musical octave gets established in the brain naturally, as soon as the reference is established. Since all the octaves are musically similar, any frequency in a higher or a lower octave can be translated to an equivalent frequency in the naturally established octave. Our brains seem to be capable of naturally converting this frequency into the first established octave without giving us a mathematical quantification but a musically correct note.

A mathematically quantified conversion can be obtained by determining the octave in which the frequency level is present and then by dividing the frequency level by  $2^{(n-1)}$  (where  $n$  is the determined octave). As an example, consider the reference as 240 Hz. How will a frequency level of 2560 Hz be translated into this octave? Clearly, the first octave extends from 240-480 Hz, second octave: 480 – 960 Hz, third octave: 960- 1920 Hz and fourth octave: 1920- 3840 Hz. So, 2560 Hz lies in the 4<sup>th</sup> octave.

Dividing 2560 by  $2^3$  yields a frequency level of 320 Hz in the first octave. So, a frequency level of 320 Hz frequency would yield the same musical effect as the frequency 2560 Hz.

Let us focus our attention on Fig. 2. The different overtones produced from the reference note (say 240 Hz) are 480, 720, 960, 1200, 1440 Hz, etc. Their conversion into the first octave yields three distinct overtones viz 300, 360, and 480 Hz that are  $5/4$ ,  $3/2$ , and 2 times higher than Shadaj, respectively; nomenclated as **Gandhar** (termed as Swayambhu Gandhar), **Pancham**, and Tar **Shadaj**, respectively.



**Fig. 2.** Overtones of a string

Let us analyse the birth of musical notes from the strings of a Tanpura—the most revered instrument of Indian classical music. The strings are generally tuned to Pancham, Tar Shadaj, and Shadaj, and each of these produces several overtones. These overtones serve as the medium for selection of other musically relevant frequency levels as follows:

- Pancham of Pancham results in a frequency level:  $3/2 * 3/2 \rightarrow 9/4 \rightarrow 9/8$  (after conversion); and termed as **Rishabh**
- Pancham of Rishabh results in a frequency level:  $9/8 * 3/2 \rightarrow 27/16$ ; and termed as **Dhaivat**
- Pancham of Gandhar results in a frequency level:  $5/4 * 3/2 \rightarrow 15/8$ ; and termed as **Nishad**
- The consonance of Pancham and Taar Shadaj results in a frequency level:  $2 * 2/3 \rightarrow 4/3$ ; and termed as **Madhyam**.

We obtain six notes being born out of Shadaj, namely, Rishabh (9/8), Gandhar (5/4), Madhyam (4/3), Pancham (3/2), Dhaivat (27/16) and Nishad (15/8). The birth of six notes from the reference note gives a license to nomenclate this reference as “Shadaj.” The term derives its origin from the Sanskrit word “Shat-j” and essentially means the one that gives birth to six new identities (musical notes). The idea of being born out of six or giving birth to six appears analogous to that of white light splitting into its component colors (on passing through a prism). Just like this phenomenon is visibly perceptible, the birth of six identical musical notes (from one note) is audibly perceptible. Our musical forefathers could comprehend this magical phenomenon, that fuelled them to nomenclate the reference note as Shadaj.

It may be argued that there may exist more than six possible musically relevant frequencies, derivable by using the mathematical jigs; thus, why the name is Shadaj?

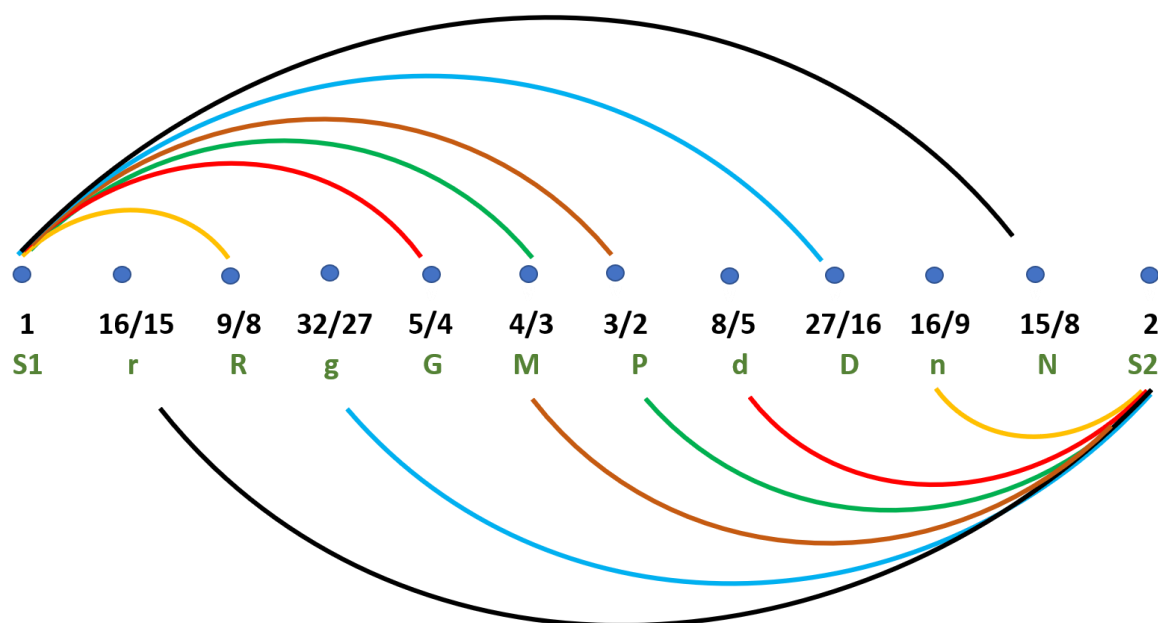
One should note that the mathematical quantification of frequencies was not available to our ancestors and has only been possible in the last few decades. In fact, there are various viewpoints on the number of “shades of the music notes (Re, Ga, Ma, Pa, Dha, and Ni)—Shrutis” that are possible in an octave. The majority [1,2,3] of them cite the number “22” and have been involved in somehow arriving at this number documented in the Bharat Natyashastra via mathematical jigs. However, no firm conclusion appears to be arrived at, and this still appears to be an open issue [4,5].

The naming of the reference note would have happened even before the time of Bharat Muni. Otherwise, he would have named it “Dwawinshati” (the one that gives birth to 22 musical levels). Despite concluding 22 musically relevant frequencies (via practical experimentation) in an octave, Bharat muni refrained from changing the name of Shadaj for the reasons unknown.

#### 4.2. Individualities of Shadaj and Tar Shadaj

Assume that we have an eight-stringed instrument, with all the eight strings tensed at different frequency levels, namely Sa, Re, Ga, Ma, Pa, Dha, Ni, and Sa (Taar). When one plays the string tuned at Sa followed by the one tuned to Pancham continuously, a unique ambiance gets accentuated known as Shadaj-Pancham bhav. However, if one plays Pancham and the last string (tuned to Tar Shadaj) continuously, this creates a different ambiance. To comprehend what conversation it is, one needs to analyze it from the reference point. Just like the musical

frequencies in any octave are comprehended by translating them into the first octave, the conversation of any two notes can be analyzed from the perspective of the reference note. The conversation of “Pancham and Tar Shadaj” is not the same as “Pancham and Shadaj” but corresponds to that of “Shadaj and Madhyam,” i.e., it accentuates “Shadaj- Madhyam” bhav. Similarly, the frequency level that is  $9/8$  times that of Shadaj is Rishabh and accentuates Shadaj- Rishabh musical conversation. But, when this frequency converses with Tar Shadaj, the musical environment is not that produced by “Shadaj and Rishabh” but that of “Shadaj- and frequency level that is  $16/9$  times Shadaj”. This frequency level is termed as Komal Nishad (a variant of Nishad). The same is true with any other frequency level, as shown in Fig 3. Similar conversations are depicted by a similar colored curve . The newly obtained frequency levels (as a result of the conversation of six notes with Tar Shadaj are designated by small alphabets as r, g, d, n, and colloquially adjectivized as Komal in relation to their successive notes).



**Fig. 3.** The similarity in musical conversations (depicted by the same colored curves).

## 5. Conclusion

The consonance of the two endpoints of an octave is a unique phenomenon that gives rise to Shadaj and Tar Shadaj. Despite being similar in their musical nature, they hold subjective relationships (mathematical and musical) with the other notes of an octave. The two Shadajs appear analogous to the identical twins, who appear the same in their physical characteristics. However, despite sharing the same genetic information, they differ in their environmental interactions and perceptions.

## Author's contributions

HS contributed to the algebraic analysis, presentation, and writing of the manuscript under the guidance of GKB.

GKB contributed to the conception, design, and evaluation of the manuscript.

### References:

1. Vidwans, V. (2016). The doctrine of shrutis in Indian music. Pune: Flame University. Recuperadode <http://www.indian-heritage.org/music/> *The Doctrine of Shruti In Indian Music-Dr. VinodVidwans*. pdf.
2. Thakur, D. S., & Thakur, D. S. The Notion of Twenty-Two Shrutis: Frequency Ratios in Hindustani Classical Music. *Resonance*, 20(06).
3. Bansod, V., & Sharma, M. Mathematical Approach for Twenty-Two Microtones: Frequency Ratios in Hindustani Classical Music & their Implementation in 22 Shruti Harmonium. *rn*, 55, 7.
4. Haldankar, S. S. (1992). Dimensions of Research in Music-A Performer's Viewpoint. *Journal of the Indian Musicological Society*, 23, 46.
5. Serra, J., Koduri, G. K., Miron, M., & Serra, X. (2011, October). Assessing the Tuning of Sung Indian Classical Music. In *ISMIR* (pp. 157-162).