

Merukhanda in Hindustani Classical Music at a Glance

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Abstract

Sound is sensational vibration that stimulates the nervous system within the ears through a medium and creates the sensation of hearing. There are two types of sounds, which are, standardized and non-standardized. Music is classified under the category of standardized sound. A musical sound is periodic or regular and its unit is called 'Nada'. Hindustani Classical Music is a popular genre, both in vocal and instrumental forms. This genre is based on the 'Ragadari style' constructed by the amalgamation of seven musical notes. A Raga is elaborated through the pleasant and melodious construction work consisting of permutations and combinations of the musical notes systematically and supremely. This construction work can be termed 'Merukhanda' (permutation-indicator of musical notes) also known as 'Khandameru'. Improvising this technique with relentless practice plays a significant role in an artiste's performance. Now, the mentioned topic is described here at a glance.

Keywords: Merukhanda, Permutation, Musical Notes, Technique

Introduction:

Permutations and combinations are the mathematical techniques used to calculate the number of possible outcomes in different ways. Musically, permutation may be defined as the arrangement of musical notes in a defined order and combination as the selection of musical notes. In Hindustani Classical Music, a total of 5040 unique combinations can be constructed from all seven musical notes. The word *Merukhanda* is a combination of two words, 'Meru' meaning mountain and 'Khanda' meaning part. *Merukhanda* can be defined as ordered combinations, i.e., permutations of a fixed set of musical notes broken up into various parts. These permutations serve as parts that help to create a mountain of music.¹

The tireless practice of this technique of improvisation in long term is beneficial to improve the unique vocalization and embellishing the elaboration of Raga.

Description of Merukhanda is stated briefly as under:

¹ Jani Rajen, Merukhand Svra Permutations Volume 1, Createspace Independent Publishing Platform, 2018, p. iii

1. History of Merukhanda:

The Merukhanda technique has been mentioned in the 13th Century Sanskrit treatise ‘Sangita Ratnakara of Sharngadeva’.

The Merukhanda technique is utilized according to the proper prominence on musical notes as required by the Raga as well as the intellectual mood of its exploration.

In the 19th Century, the Merukhanda techniques were expanded within the Bhendibazaar Gharana. Ustad Aman Ali Khan (1888-1953) applied this technique for extended Alaps and Sargam patterns. Ustad Abdul Wahid Khan (1871-1949) of Kirana Gharana mastered the practice of merukhandi passages in his performance. Later, Ustad Amir Khan (1912-1974) of Indore Gharana introduced a new style of manipulating Merukhanda practices. He used this technique intellectually in his performances to make them unique and compelling.

Mathematical analysis of Merukhanda:

The implication of mathematics for the construction of Merukhanda is described below:

2.1 Combination method (*order does not matter*):

Formula: ${}^nC_r = \frac{n!}{n! \times (n-r)!}$

(where ‘n’ = total number of musical notes, ‘r’ = selected musical notes within ‘n’ and ‘!’ sign denotes factorial)

For example, we have a total of seven musical notes. Now, in how many ways can we choose five notes from the group. Every probable arrangement can be a combination.

The combination of ‘r’ (here the value is 5) from the available ‘n’ (here the value is 7) is used here as per the formula:

$${}^7C_5 = \frac{7!}{5! \times (7-5)!} = \frac{7 \times 6 \times 5!}{5! \times 2!} = \frac{7 \times 6}{2!} = \frac{42}{2 \times 1} = 21$$

Hence, we get 21 possible combinations.

2.2 Permutation method (*order matters*):

Formula: ${}^nP_r = \frac{n!}{n! \times (n-r)!}$

(where ‘n’ = total number of musical notes, ‘r’ = selected musical notes within ‘n’ and ‘!’ sign denotes factorial)

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For example, we have a total of seven musical notes. Now, in how many ways can we choose five notes from the group. Every ordered combination will be a permutation.

The permutation of 'r' (here the value is 5) from the available 'n' (here the value is 7) is used here as per the formula:

$${}^7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 2520$$

Hence, we get the permutation value as **2520**.

The above values of 'n' and 'r' reveal that the number of permutations is greater than the number of combinations.

2.3 Construction of Merukhanda:

The mathematical graph of Merukhanda as mentioned by Sharngdeva in his text 'Sangita Ratnakara' helps to solve two musical problems collectively known as '*Nashta-Uddishta*'.

Merukhanda (permutation-indicator)²

Sa	Re	Ga	Ma	Pa	Dha	Ni
1	0	0	0	0	0	0
	1	2	6	24	120	720
		4	12	48	240	1440
			18	72	360	2160
				96	480	2880
					600	3600
						4320

Nashta refers to the method for finding out the 'missing note series' and *Uddishta* refers to the method for finding out the 'indicated note series'.

2.4 The number of permutational note-series in each type derived from the Merukhanda (permutation-indicator)³

² Shringy R. K. and Sharma Prem Lata, Sangitaratnakara of Sarngadeva Text and English Translation Vol. I, Munshiram Manoharlal Publishers Pvt. Ltd., Fifth edition 2013, Chart 23, p. 209

³ Shringy R. K. and Sharma Prem Lata, Sangitaratnakara of Sarngadeva Text and English Translation Vol. I, Munshiram Manoharlal Publishers Pvt. Ltd., Fifth edition 2013, Chart 27, p. 222

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The final note of the series in its original order	Name of the type of the series	Column squares		Figures of the last column squares		Total of last squares	
		S. No. of col/s.	No. of square/s	S. No. of the last col. sq.	Figure/s indicated	Figure/s added	Total
Sa	Monotone series	1	1	1 st	1	1	1
Re	Bitonic series	2	2	2 nd	1	1+1	2
Ga	Tritonic series	3	3	3 rd	4	1+1+4	6
Ma	Tetratonic series	4	4	4 th	18	1+1+4+18	24
Pa	Pentatonic series	5	5	5 th	96	1+1+4+18+96	120
Dha	Hexatonic series	6	6	6 th	600	1+1+4+18+96+600	720
Ni	Heptatonic series	7	7	7 th	4320	1+1+4+18+96+600+4320	5040

2.4.1 Permutations of a fixed set of musical notes:

The *Saptaka Svra Ashtaka Krama* is defined as a term signifying an arrangement of the seven musical notes (svaras) in an octave format.⁴ This concept is applied in the permutation of musical note series.

2.4.1.1 One musical note permutation:

1 fixed musical note from 8 groups Sa, Re, Ga, Ma, Pa, Dha, Ni and Sa

here n is 1, then 1! (factorial) is $1 \times 1 = 1$ permutation

For example, when one musical note 'Sa' is taken, the permutation will be 1, i.e., Sa

8 groups, each having 1 permutation will make a total of $8 \times 1 = 8$ permutations.

2.4.1.2 Two musical notes permutation:

2 fixed musical notes from the 7 groups

Sa Re, Re Ga, Ga Ma, Ma Pa, Pa Dha, Dha Ni and Ni Sa

here n is 2, then 2! (factorial) is $2 \times 1 = 2$ permutations

For example, when two musical notes 'Sa' and 'Re' are taken, the permutations will be 2, i.e. Sa Re and Re Sa

7 groups, each having 2 permutations will make a total of $7 \times 2 = 14$ permutations.

⁴ Jani Rajen, Merukhand Svra Permutations Volume 1, Createspace Independent Publishing Platform, 2018. p. iv

2.4.1.3 Three musical notes permutation:

3 fixed musical notes from the 6 groups

Sa Re Ga, Re Ga Ma, Ga Ma Pa, Ma Pa Dha, Pa Dha Ni and Dha Ni Sa

here n is 3, then 3! (factorial) is $3 \times 2 \times 1 = 6$ permutations

For example, when three musical notes 'Sa', 'Re' and 'Ga' are taken, the permutations will be 6, i.e. Sa Re Ga, Sa Ga Re, Re Sa Ga, Re Ga Sa, Ga Sa Re and Ga Re Sa

6 groups, each having 6 permutations will make a total of $6 \times 6 = 36$ permutations.

2.4.1.4 Four musical notes permutation:

4 fixed musical notes from the 5 groups

Sa Re Ga Ma, Re Ga Ma Pa, Ga Ma Pa Dha, Ma Pa Dha Ni and Pa Dha Ni Sa

here n is 4, then 4! (factorial) is $4 \times 3 \times 2 \times 1 = 24$ permutations

For example, when four musical notes 'Sa', 'Re', 'Ga' and 'Ma' are taken, the permutations will be 24, i.e.

Sa Re Ga Ma,	Sa Re Ma Ga,	Sa Ga Re Ma,	Sa Ga Ma Re,
Sa Ma Re Ga,	Sa Ma Ga Re,	Re Sa Ga Ma,	Re Sa Ma Ga,
Re Ga Sa Ma,	Re Ga Ma Sa,	Re Ma Sa Ga,	Re Ma Ga Sa,
Ga Sa Re Ma,	Ga Sa Ma Re,	Ga Re Sa Ma,	Ga Re Ma Sa,
Ga Ma Sa Re,	Ga Ma Re Sa,	Ma Sa Re Ga,	Ma Sa Ga Re,
Ma Re Sa Ga,	Ma Re Ga Sa,	Ma Ga Sa Re,	Ma Ga Re Sa

5 groups, each having 24 permutations will make a total of $5 \times 24 = 120$ permutations.

2.4.1.5 Five musical notes permutation:

5 fixed musical notes from the 4 groups

Sa Re Ga Ma Pa, Re Ga Ma Pa Dha, Ga Ma Pa Dha Ni and Ma Pa Dha Ni Sa

here n is 5, then 5! (factorial) is $5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations

For example, when five musical notes 'Sa', 'Re', 'Ga', 'Ma' and 'Pa' are taken, the permutations will be 120.

Some of the permutational note-series among 120 permutations are appended below:

Sa Re Ga Ma Pa,	Sa Re Ga Pa Ma,	Sa Re Ma Ga Pa,	Sa Re Ma Pa Ga,
Sa Re Pa Ga Ma,	Sa Re Pa Ma Ga,	Sa Ga Re Ma Pa,	Sa Ga Re Pa Ma,
Sa Ga Ma Re Pa,	Sa Ga Ma Pa Re,	Sa Ga Pa Re Ma,	Sa Ga Pa Ma Re;

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Re Sa Ga Ma Pa, Re Sa Ga Pa Ma, Re Sa Ma Ga Pa, Re Sa Ma Pa Ga,
 Re Sa Pa Ga Ma, Re Sa Pa Ma Ga, Re Ga Sa Ma Pa, Re Ga Sa Pa Ma,
 Re Ga Ma Sa Pa, Re Ga Ma Pa Sa, Re Ga Pa Sa Ma, Re Ga Pa Ma Sa;
 Ga Sa Re Ma Pa, Ga Sa Re Pa Ma, Ga Sa Ma Re Pa, Ga Sa Ma Pa Re,
 Ga Sa Pa Re Ma, Ga Sa Pa Ma Re, Ga Re Sa Ma Pa, Ga Re Sa Pa Ma,
 Ga Re Ma Sa Pa, Ga Re Ma Pa Sa, Ga Re Pa Sa Ma, Ga Re Pa Ma Sa;
 Ma Sa Re Ga Pa, Ma Sa Re Pa Ga, Ma Sa Ga Re Pa, Ma Sa Ga Pa Re,
 Ma Sa Pa Re Ga, Ma Sa Pa Ga Re, Ma Re Sa Ga Pa, Ma Re Sa Pa Ga,
 Ma Re Ga Sa Pa, Ma Re Ga Pa Sa, Ma Re Pa Sa Ga, Ma Re Pa Ga Sa;
 and so on

4 groups, each having 120 permutations will make a total of $4 \times 120 = 480$ permutations.

2.4.1.6 Six musical notes permutation:

6 fixed musical notes from the 3 groups

Sa Re Ga Ma Pa Dha, Re Ga Ma Pa Dha Ni and Ga Ma Pa Dha Ni Sa

here n is 6, then 6! (factorial) is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ permutations

For example, when six musical notes 'Sa', 'Re', 'Ga', 'Ma', 'Pa' and 'Dha' are taken, the permutations will be 720.

Some of the permutational note-series among 720 permutations are appended below:

Sa Re Ga Ma Pa Dha, Sa Re Ga Ma Dha Pa, Sa Re Ga Pa Ma Dha,
 Sa Re Ga Pa Dha Ma, Sa Re Ga Dha Ma Pa, Sa Re Ga Dha Pa Ma,
 Sa Re Ma Ga Pa Dha, Sa Re Ma Ga Dha Pa, Sa Re Ma Pa Ga Dha;
 Re Sa Ga Ma Pa Dha, Re Sa Ga Ma Dha Pa, Re Sa Ga Pa Ma Dha,
 Re Sa Ga Pa Dha Ma, Re Sa Ga Dha Ma Pa, Re Sa Ga Dha Pa Ma,
 Re Sa Ma Ga Pa Dha, Re Sa Ma Ga Dha Pa, Re Sa Ma Pa Ga Dha;
 and so on

3 groups, each having 720 permutations will make a total of $3 \times 720 = 2,160$ permutations.

2.4.1.7 Seven musical notes permutation:

7 fixed musical notes from the 2 groups

Sa Re Ga Ma Pa Dha Ni and Re Ga Ma Pa Dha Ni Sa

here n is 7, then 7! (factorial) is $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$ permutations

Some of the permutational note-series among the 5,040 permutations are appended below:

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Sa Re Ga Ma Pa Dha Ni, Sa Re Ga Ma Pa Ni Dha, Sa Re Ga Ma Dha Pa Ni,
 Sa Re Ga Ma Dha Ni Pa, Sa Re Ga Ma Ni Pa Dha, Sa Re Ga Ma Ni Dha Pa,
 Sa Re Ga Pa Ma Dha Ni, Sa Re Ga Pa Ma Ni Dha, Sa Re Ga Pa Dha Ma Ni,
 Sa Re Ga Pa Dha Ni Ma, Sa Re Ga Pa Ni Ma Dha, Sa Re Ga Pa Ni Dha Ma,
 Sa Re Ga Dha Ma Pa Ni, Sa Re Ga Dha Ma Ni Pa, Sa Re Ga Dha Pa Ma Ni;
 and so, on

2 groups, each having 5,040 permutations will make a total of $2 \times 5,040 = 10,080$ permutations.

Hence, the grand total of the above musical note permutational series will be:

$$8+14+36+120+480+2,160+10,080 = 12, 898$$

2.4.2 Examples of Merukhanda-Alankara patterns used in Ragas:**2.4.2.1 Raga Malkauns (Pentatonic Raga):**

Aroha: Sa Ga Ma Dha Ni Sa

Avaroha: Sa Ni Dha Ma Ga Sa

The possible permutational note-series in this Raga will be

$${}^5P_5 = \frac{5!}{(5-5)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{0! \text{ (Value of } 0! = 1)} = 120$$

Some of the Merukhanda-Alankara patterns are as under:

Sa Ga Ma Dha Ni Sa Ga Ma Ni Dha Sa Ga Dha Ma Ni Sa Ga Dha Ni Ma
Ga Ma Dha Ni Sa Ga Ma Dha Sa Ni Ga Ma Ni Dha Sa Ga Ma Ni Sa Dha
 Ma Dha Ni Sa Ga Ma Dha Ni Ga Sa Ma Dha Sa Ni Ga Ma Dha Sa Ga Ni
Dha Ni Sa Ga Ma Dha Ni Sa Ma Ga Dha Ni Ga Sa Ma Dha Ni Ga Ma Sa
Ni Sa Ga Ma Dha Ni Sa Ga Dha Ma Ni Sa Ma Ga Dha Ni Sa Ma Dha Ga
 and so on

2.4.2.2 Raga Janasammohini (Hexatonic Raga):

Aroha: Sa Re Ga Pa Dha Ni Sa

Avaroha: Sa Ni Dha Pa Ga Re Sa

The possible permutational note-series in this Raga will be

$${}^6P_6 = \frac{6!}{(6-6)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{0! \text{ (Value of } 0! = 1)} = 720$$

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Some of the Merukhanda-Alankara patterns are as under:

Sa Re Ga Pa Dha <u>Ni</u>	Sa Re Ga Pa <u>Ni</u> Dha	Sa Re Ga Dha Pa <u>Ni</u>
Re Ga Pa Dha <u>Ni</u> Sa	Re Ga Pa Dha Sa <u>Ni</u>	Re Ga Pa <u>Ni</u> Dha Sa
Ga Pa Dha <u>Ni</u> Sa Re	Ga Pa Dha <u>Ni</u> Re Sa	Ga Pa Dha Sa <u>Ni</u> Re
Pa Dha <u>Ni</u> Sa Re Ga	Pa Dha <u>Ni</u> Sa Ga Re	Pa Dha <u>Ni</u> Re Sa Ga
Dha <u>Ni</u> Sa Re Ga Pa	Dha <u>Ni</u> Sa Re Pa Ga	Dha <u>Ni</u> Sa Ga Re Pa
<u>Ni</u> Sa Re Ga Pa Dha	<u>Ni</u> Sa Re Ga Dha Pa	<u>Ni</u> Sa Re Pa Ga Dha

and so on.

2.4.2.3 Raga Basant Mukhari (Heptatonic Raga):

Aroha: Sa Re Ga Ma Pa Dha Ni Sa Avaroha: Sa Ni Dha Pa Ma Ga Re

The possible permutational note-series in this Raga will be

$${}^7P_7 = \frac{7!}{(7-7)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{0! \text{ (value of } 0! = 1)} = 5,040$$

Some of the Merukhanda-Alankara patterns are as under:

Sa <u>Re</u> Ga Ma Pa <u>Dha</u> <u>Ni</u>	Sa <u>Re</u> Ga Ma Pa <u>Ni</u> <u>Dha</u>	Sa <u>Re</u> Ga Ma <u>Dha</u> Pa <u>Ni</u>
<u>Re</u> Ga Ma Pa <u>Dha</u> <u>Ni</u> Sa	<u>Re</u> Ga Ma Pa <u>Dha</u> Sa <u>Ni</u>	<u>Re</u> Ga Ma Pa <u>Ni</u> <u>Dha</u> Sa
Ga Ma Pa <u>Dha</u> <u>Ni</u> Sa <u>Re</u>	Ga Ma Pa <u>Dha</u> <u>Ni</u> Re Sa	Ga Ma Pa <u>Dha</u> Sa <u>Ni</u> <u>Re</u>
Ma Pa <u>Dha</u> <u>Ni</u> Sa <u>Re</u> Ga	Ma Pa <u>Dha</u> <u>Ni</u> Sa Ga <u>Re</u>	Ma Pa <u>Dha</u> <u>Ni</u> <u>Re</u> Sa Ga
Pa <u>Dha</u> <u>Ni</u> Sa <u>Re</u> Ga Ma	Pa <u>Dha</u> <u>Ni</u> Sa <u>Re</u> Ma Ga	Pa <u>Dha</u> <u>Ni</u> Sa Ga <u>Re</u> Ma
<u>Dha</u> <u>Ni</u> Sa <u>Re</u> Ga Ma Pa	<u>Dha</u> <u>Ni</u> Sa <u>Re</u> Ga Pa Ma	<u>Dha</u> <u>Ni</u> Sa <u>Re</u> Ma Ga Pa
<u>Ni</u> Sa <u>Re</u> Ga Ma Pa <u>Dha</u>	<u>Ni</u> Sa <u>Re</u> Ga Ma <u>Dha</u> Pa	<u>Ni</u> Sa <u>Re</u> Ga Pa Ma <u>Dha</u>

and so on.

3. Application and Benefits of Merukhanda:

The application and benefits of practicing the Merukhanda techniques are as follows:

- It helps to stabilize the vocal range and improves the voice texture.
- It is beneficial for the music learners to undertake *Riyaaz* systematically.
- It helps to improve thinking ability and develops creativity in the mind of an artist.
- It helps to create unique framing of Alaps, Sargams and Tana patterns according to the proper emphasis on musical notes.

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- Application of the Merukhanda techniques in the gayaki (stylistic approach to *raga* development)⁵ of an artist makes his/her performance attractive and engrossing.
- It benefits all listeners to easily grasp the intricate patterns of Raga/s presented by an artist.

4. Conclusion:

In mathematical terms, 'Merukhanda' is the permutation of musical notes. Practicing this technique from the primary level with perseverance makes a fearless musical sense of non-repeating note patterns during the performance. The relentless practice of this technique not only builds up confidence but also eliminates mental hesitancy during the performance. The practice of complicated Swara-Alankaras by the systematic use of Merukhanda grows up a sense of perfection on every musical note.

Practicing Merukhanda-Alankaras might be possible in the following ways:

- Patterns created with notes of a Raga
- Patterns created with the musical notes that are not adjacent to each other
- Arrangements created with partial patterns of musical notes

The rigorous practice of Merukhanda techniques makes the mind of an artiste playful while creating note patterns. Last but not the least, proper guidance from a Guru is essential for learning the Merukhanda techniques and their usage in Raga exploration.

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⁵ Nagarkar Samarth, *Raga Sangeet: Understanding Hindustani Classical Vocal Music*, Chhandayan Inc., 1st Edition: 2013, p. 108